TEMPERATURE AND RELAXATION DIFFERENCE FOR A MAGNETICALLY ACTIVE PLASMA IN THE QUASI-LINEAR APPROXIMATION TAKING COLLISIONS INTO ACCOUNT

G. Z. Machabeli

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 2, pp. 47–51, 1969

An equation is derived for longitudinal waves in a plasma in a constant magnetic field in the quasi-linear approximation with account for collisions. The contribution made by the interaction of waves with resonant particles to the temperature relaxation of the two-component plasma is calculated. The difference between the electron and ion temperatures is found for the steady state, as is the difference between the temperature components perpendicular and parallel to the magnetic field for particles of one kind in the electric field of the wave.

1. The quasi-linear approximation, of interest for slightly turbulent phenomena, describes effects which have been interpreted as the interactions of resonant particles with waves for the case in which the plasma-oscillation energy is much smaller than the random-motion energy of the particles and much greater than the thermal-noise energy in the collective degrees of freedom.

In this paper we consider the quasi-linear approximation for a magnetically active plasma taking collisions among charged particles into account. The condition

$$H \ll 2c\,(\pi m_a n_a)^{1/2} \tag{1.1}$$

is imposed on the magnetic field, where n_a is the number of particles of type a per unit volume. Condition (1.1) means that the Larmor radius for particles of type a is much greater than the Debye radius for the same particles. The effect of the magnetic field on particle collisions can thus be neglected.

In treating equations with a self-consistent field, we can single out two ranges: the shortwave range (the collision range) and the long-wave range (the radiation range) [1]. The shortwave range was neglected in the quasi-linear-approximation treatment in [1] since short-wave excitations are rapidly damped in the self-consistent-field approximation. To account for both the self-consistent field and dissipative effects, however, we follow [2], basing the discussion on equations in which both the self-consistent field and collisions are taken into account.

For a magnetically active plasma we have

$$\frac{\partial f_a}{\partial t} + \mathbf{v}\frac{\partial f_a}{\partial \mathbf{q}} + e\mathbf{E}\frac{\partial f_a}{\partial \mathbf{p}_a} + \frac{e_a}{c}[\mathbf{vH}]\frac{\partial f_a}{\partial \mathbf{p}_a} = S_a\,. \tag{1.2}$$

We break up the particle distribution function in the following manner:

$$f_a = f_{0a} + f_{1a}\,, \tag{1.3}$$

where f_{0a} is the slowly changing distribution function and f_{1a} is the rapidly changing distribution function. Imposing the condition

$$\frac{eE}{m_a v_a \omega} \ll 1$$

on the average field, and substituting (1.3) into (1.2), we find two equations for the "background" and the "pulsation:"

$$\frac{\partial f_{0a}}{\partial t} + \mathbf{v}\frac{\partial f_{0a}}{\partial \mathbf{q}} + \left\langle e\mathbf{E}\frac{\partial f_{1a}}{\partial \mathbf{p}_a}\right\rangle + \frac{e}{c}[\mathbf{vH}]\frac{\partial f_{0a}}{\partial \mathbf{p}_a} = S_{0a}\,, \tag{1.4}$$

$$\frac{\partial f_{1a}}{\partial t} + \mathbf{v}\frac{\partial f_{1a}}{\partial \mathbf{q}} + \frac{e}{c}[\mathbf{vH}]\frac{\partial f_{1a}}{\partial p_a} + e\mathbf{E}\frac{\partial f_{0a}}{\partial \mathbf{p}_a} = S_{1a} \tag{1.5}$$

[the angle brackets in the third term of (1.4) denote an averaging over the rapid pulsations]. Here S_{0a} is an integral which describes the collisional contribution to the "background" distribution function and which is written in the Landau form for a Coulomb plasma; S_{1a}, the collision integral for the rapidly changing distribution function, is written in the following manner [2]:

$$S_{1a} = \nu_a(\mathbf{v}) f_{1a} \,. \tag{1.6}$$

Here $\nu_a(\mathbf{v})$ is the reverse relaxation time due to collisions contributing to the rapidly changing part of the distribution function, and $\nu_a(\mathbf{v})$ is a function of the velocity; for a slightly nonequilibrium plasma, however, we assume ν_a to be an average over momenta of the effective collision frequency, given for cold plasmas in [3,4]. For a hot plasma we assume ν_a is some parameter approximately equal to the electron-ion collision frequency.

We can use Eq. (1.6) to solve Eq. (1.5); carrying out a Fourier coordinate and time transformation of this solution and converting to the cylindrical coordinates ($v_x = v_\perp \cos\varphi$, $v_y = v_\perp \sin\varphi$, $v_z = v_z$; $k_x = k_\perp \cos\psi$, $k_y = k_\perp \sin\psi$, $k_z = k_z$), we find after some calculation that

$$f_{1a}(\omega, \mathbf{k}, \mathbf{p}_a) = \sum_{n=-\infty}^{+\infty} J_n^2\left(\frac{k_\perp v_\perp}{\Omega_a}\right) \frac{e_a \Phi}{\omega - k_z v_z - n\Omega_a + i\nu_a} \times \\ \times \left[k_z \frac{\partial}{\partial p_{az}} + \frac{n\Omega_a}{v_\perp} \frac{\partial}{\partial p_{a\perp}}\right] f_{0a} \,. \tag{1.7}$$

Here $\Omega_a = eH/m_a c$, $E = -ik\Phi$, $J_n(k_\perp v_\perp/\Omega_a)$ is the Bessel function, and the subscript $\perp$ denotes a quantity whose sense is perpendicular to the magnetic field.

Substituting (1.7) into (1.5), and averaging over the angle φ, we find the isotropic "background" function to be

$$\frac{\partial f_{0a}}{\partial t} - \sum_{n=-\infty}^{+\infty} \left[k_z \frac{\partial}{\partial p_{az}} + \frac{n\Omega_a}{v_\perp} k_\perp \left(\frac{\partial}{\partial p_{a\perp}} + \frac{1}{p_\perp}\right)\right] \frac{1}{2} \frac{I_n^2(k_\perp v_\perp/\Omega_a)(e\Phi)^2 \nu_a}{(\omega - k_z v_z - n\Omega_a)^2 + \nu_a^2} \times \\ \times \left[k_z \frac{\partial}{\partial p_{az}} + \frac{n\Omega_a}{v_\perp} \frac{\partial}{\partial p_{a\perp}}\right] f_{0a} + \frac{e}{c} [\mathbf{vH}] \frac{\partial f_{0a}}{\partial \mathbf{p}_a} = S_{0a} \,. \tag{1.8}$$

Equation (1.8) is the solution sought for a Coulombic, magnetically active plasma with an account of collisions in the quasi-linear approximation.

In the derivation of the quasi-linear-approximation equations [5, 6], the plasma-wave phases were assumed random and it was assumed that these phases correlate with each other over a time much smaller than the relaxation time of the function f_{0a}. Here, as in [2], we do not assume random phases, and Eq. (1.8) was derived for a plane wave. In the case $H = 0$, Eq. (1.8) is the same as that obtained in the quasi-linear approximation with account for collisions but without a magnetic field [2].

2. Landau [7] and Kogan [8] have treated temperature relaxation in a nonisothermal plasma. Only binary collisions of particles were taken into account. Ramazashvili et al. [9] have shown that an account of long-range interactions between particles and plasma waves makes an important contribution to the temperature relaxation rate for a Coulombic plasma. The long-range interactions, like the binary interactions, were incorporated in the collision integral in [9], yielding a term describing the interaction of thermal noise with the particles. In this paper, we incorporate only binary collisions in the collision integral in Eq. (1.8); we incorporate the interaction between resonant particles and plasma waves whose amplitude is consistent with the quasi-linear approximation on the left-hand side of the equation. For this purpose, we first find the energy balance equation from Eq. (1.8).

We write the particle distribution function for particles of type a in the Maxwell form:

$$f_{0a} = \frac{n_a}{\pi^{3/2} v_{az} v_{a\perp}^2} \exp\left(-\frac{v_z^2}{v_{az}^2} - \frac{v_\perp^2}{v_{a\perp}^2}\right), \tag{2.1}$$

where v_a is the thermal velocity of the particles. We multiply Eq. (1.8) by $m_a v_z^2/2$, insert (2.1), and integrate over dv; the result is

$$\frac{1}{2} n_a \frac{dT_{az}}{dt} - n_a \sum_{n=-\infty}^{+\infty} \exp\left(-\frac{k_\perp^2 v_{a\perp}^2}{2\Omega_a^2}\right) I_n\left(\frac{k_\perp^2 v_{a\perp}^2}{2\Omega_a^2}\right) \times$$
$$\times \frac{(e\Phi)^2}{m_a} \left\{ \frac{\nu_a}{v_{a\perp}^2} \left(1 - \pi^{1/2}\left[2x_a \mathrm{Im} W(z_a) - \frac{x_a^2 - y_a^2}{y_a} \mathrm{Re} W(z_a)\right]\right) + \right.$$
$$\left. + \pi^{1/2} \frac{n\Omega_a}{v_{a\perp}^2} (x_a \mathrm{Re} W(z_a) - y_a \mathrm{Im} W(z_a)) \right\} = \int {}^1\!/_2 m_a v_{az}^2 S_{0a}\, d\mathbf{v}\,, \qquad (2.2)$$

where

$$T_{az} = \int \frac{1}{2} m_a v_z^2 f_{0a}\, d\mathbf{v}$$

is the temperature for particles of type a;

$$W(z_a) = \exp(-z^2)\left[1 + \frac{2i}{\pi^{1/2}} \int_0^z \exp(t^2)\, dt\right]$$

is the probability integral;

$$z_a = x_a + y_a, \; x_a = \frac{\omega - n\Omega_a}{k_z v_{az}}, \quad y_a = \frac{\nu_a}{k_z v_{az}}, \text{ and } I_n\left(\frac{k_\perp^2 v_{a\perp}^2}{2\Omega_a^2}\right)$$

is the modified Bessel function.

For further discussion, we consider separately the two limiting cases of hot and cold plasmas.

a. We first consider a cold plasma, for which

$$\frac{(\omega - n\Omega_a)^2}{k_z^2 v_{az}^2} \gg 1\,, \qquad (2.3)$$

and in which we assume a rather large magnetic field:

$$\frac{k_\perp^2 v_{a\perp}^2}{2\Omega_a^2} \ll 1\,. \qquad (2.4)$$

Because of condition (2.4), we can restrict the expansion of the Bessel function to terms with n = 0. We neglect resonances at higher harmonics n = ± 1, ±2, ...

For the case of a two-component electron-ion plasma, for which $T_e \neq T_i$ and $T_{az} = T_{a\perp} (a = e, i)$, we use the asymptotic expression for the probability integral for the case of a large argument; taking inequality (2.4) into account, we find from (2.2) that

$$\frac{3}{2} \frac{dT_e}{dt} = \frac{(e\Phi)^2 k_z^2}{2m_e \omega^2} \nu_e - \frac{3}{2} \frac{T_e - T_i}{\tau_{ei}^T}\,. \qquad (2.5)$$

Here τ^T_{ei} is the temperature relaxation time due to electron-ion collisions. In deriving (2.5) we have neglected ν_a^2 in comparison with ω^2.

The first term on the right of (2.5) describes the contribution to the temperature change due to the interaction of the wave with resonant particles; the second term describes the contribution due to binary Coulombic collisions (electron-ion collisions).

For the ions we have an equation analogous to Eq. (2.5), except that the quasi-linear term is a factor of m_e/m_i smaller than the analogous term in Eq. (2.5). Neglecting this term, we find

$$\frac{3}{2} \frac{dT_i}{dt} = -\frac{3}{2} \frac{T_i - T_e}{\tau_{ei}^T}\,. \qquad (2.6)$$

Since the damping decrement in quasi-linear theory is small in comparison with the wave frequency, we assume the amplitude of the wave field to be independent of the time. Under this assumption, we find from Eqs. (2.5) and (2.6), using the notation $\Delta T(t) = T_e(t) - T_i(t)$, the following expression for the relaxation of the electron-ion temperatures:

$$\Delta T(t) = \Delta T(t=0) \exp\left(-\frac{t}{\tau_{ei}^T}\right) + \frac{(e\Phi)^2 k_z^2 v_e \tau_{ei}^T}{3 m_e \omega^2} \left(1 - \exp\left(-\frac{t}{\tau_{ei}^T}\right)\right). \tag{2.7}$$

Clearly, the contribution in (2.7) from the interaction between the wave and the particles prevents temperature relaxation. This happens because the wave interacts actively with plasma electrons, while the transfer of energy to ions by the wave is hindered because of the large ion mass.

When there is a constant source exciting high-frequency waves, the plasma relaxes to a steady state in which electron heating occurs. In the steady state, we have

$$T_e - T_i = \frac{(e\Phi)^2 k_z^2 \tau_{ei}^T}{3 m_e \omega^2}, \tag{2.8}$$

which is the same as the corresponding equation in [10], obtained for a nonmagnetic plasma. The agreement is a consequence of the fact that the magnetic field has no effect parallel to itself.

b. For the case of a hot plasma, for which we have

$$k_z v_i \ll \omega \ll k_z v_{ez} \tag{2.9}$$

(the subscript i refers to plasma ions), low-frequency waves satisfying condition (2.4) may propagate in the plasma. For such waves we have

$$\begin{aligned} \frac{3}{2}\frac{dT_e}{dt} - \pi^{1/2}\frac{k_z^2 (e\Phi)^2 \omega^2}{m_e k_z^3 v_{ez}^3} &= -\frac{3}{2}\frac{T_e - T_i}{\tau_{ei}^T}, \\ \frac{3}{2}\frac{dT_i}{dt} - \frac{k_z^2 (e\Phi)^2 v_i}{2 m_i \omega^2} &= -\frac{3}{2}\frac{T_i - T_e}{\tau_{ei}^T}. \end{aligned} \tag{2.10}$$

Equations (2.10) describe the relaxation of the electron-ion temperatures in the following manner:

$$\begin{aligned} \Delta T(t) = \Delta T(t=0) \exp\left(-\frac{t}{\tau_{ei}^T}\right) + \\ + \frac{2}{3}\left(\pi^{1/2}\frac{\omega^2}{m_e k_z^3 v_{ez}^3} + \frac{v_i}{2 m_i \omega^2}\right) k_z^2 (e\Phi)^2 \tau_{ei}^T \left(1 - \exp\left(-\frac{t}{\tau_{ei}^T}\right)\right). \end{aligned} \tag{2.11}$$

For waves satisfying condition (2.9), therefore, the plasma relaxes to a steady state with an effective electron temperature

$$T_e = T_i + \frac{2}{3}\left(\frac{\pi^{1/2}\omega^2}{m_e k_z^3 v_{ez}^3} + \frac{v_i}{2 m_i \omega^2}\right) k_z^2 (e\Phi)^2 \tau_{ei}^T. \tag{2.12}$$

For a nonisotropic temperature distribution, i.e., when $T_{\alpha z} \neq T_{\alpha\perp}$, $T_e = T_i = T$, we find the following for high-frequency waves satisfying Eqs. (2.3) and (2.4), under the assumption $|T_\perp - T_z| \ll T$:

$$T_z - T_\perp = \frac{(e\Phi)^2 k_z^2 v_e \tau}{m_e \omega^2}, \qquad \tau = \frac{4}{5}\left(\frac{m_e T^3}{\pi}\right)^{1/2} \frac{1}{e^4 L}. \tag{2.13}$$

It follows that the z-th temperature component increases. The increase in $T_\perp$ is due to the small parameter (2.4), and we neglect it.

Significantly, the effect of collisions contributing to the rapidly changing part of the electron distribution function is significant only in the high-frequency case.

In conclusion, the author thanks Yu. L. Klimontovich for suggesting the topic, and L. M. Gorbunov, V. V. Logvinov, and R. R. Ramazashvili for interest in the study.

REFERENCES

1. Yu. L. Klimontovich, Statistical Theory for Nonequilibrium Phenomena in a Plasma [in Russian], Izd-vo MGU, Moscow, 1964.

2. Y. L. Klimontovich, "The statistical theory of nonequilibrium processes in a plasma," Internat. Ser. Monogr. Natural Philos., Pergamon Press, Oxford-London-New York-Sydney-Paris-Braunschweig, 1967.

3. V. P. Silin, "High-frequency dielectric constant of a plasma," ZhÉTF, vol. 41, no. 3 (9), 1963.

4. V. V. Logvinov, "Relaxation in a Coulomb quasi-equilibrium plasma," Vestnik MGU, fizika, astronomiya [Moscow University Physics Bulletin], no. 1, 1967.

5. A. A. Vedenov, E. P. Velikhov, "Quasi-linear approximation in the kinetics of a low-density plasma," ZhÉTF, vol. 43, no. 3 (9), 1962.

6. A. A. Vedenov, "Quasi-linear plasma theory," Atomn. energiya, vol. 13, no. 1, pp. 5–24, 1962.

7. L. D. Landau, "Kinetic equation for the case of a Coulombic interaction," ZhÉTF, vol. 7, no. 2, 1937.

8. V. N. Kogan, "Equalizing rate of charged-particle temperatures in a plasma," collection: Plasma Physics and the Problem of Controlled Thermonuclear Reactions, Vol. I [in Russian], Izd-vo AN SSSR, 1958.

9. R. R. Ramazashvili, A. A. Rukhadze, and V. P. Silin, "Equalizing rate of charged-particle temperatures in a plasma," ZhÉTF, vol. 43, no. 4 (10), 1962.

10. Yu. L. Klimontovich and V. V. Logvinov, "Steady-state solutions for the quasi-linear-approximation equations in a plasma with collisions," PMTF [Journal of Applied Mechanics and Technical Physics], vol. 2, p. 35, 1967.

21 October 1968

Tbilisi